***Unit 4***

**Chapter 1 – Sequences and Series**

**1.1 – Arithmetic Sequences**

Arithmetic Sequence – a sequence in which the difference of the numbers is always the same.

Common Difference – The number you add or subtract to get the next number in the sequence

FORMULA:

tn = t1 + (n – 1)d

n = how many terms

tn = the last term, general term or nth term

d – common difference

t1 = first term

Example:

1. Many Factors affect the growth of a child. Medical and health officials encourage parents to keep track of their child’s growth. The general guideline for the growth in height of a child between the ages of 3 years and 10 years is an average increase of 5 cm per year. Suppose a child was 70 cm tall at age 3.
   1. Write the general term that you could use to estimate what the child’s height will be at any age between 3 and 10.
   2. How tall is the child expected to be at age 10.
2. Carpenter ants are large, usually black ants that make their colonies in wood. Although often considered to be pests around the home, carpenter ants play a significant role in a forested ecosystem. Carpenter ants begin with a parent colony. When this colony is well established, they form satellite colonies consisting of only the workers. An established colony may have as many as 3000 ants. Suppose the growth of the colony produces an arithmetic sequence in which the number of ants increases by approximately 80 ants each month. Beginning with 40 ants, how many months would it take for the ant population to reach 3000?
3. Jonathon has been given the job of stacking cans in a similar design to that of the cereal boxes (shown on page 14). The number of cans in the rows produce an arithmetic sequence. The top three rows are shown. There are 14 cans in the 8th row from the bottom and 10 cans in the 12th row from the bottom. Determine t1, d, and tn, for the arithmetic sequence.
4. What is the charge for 10 h if the furnace technician charges $45 for the house call plus $46 per hour?

**1.2 – Arithmetic Series**

Arithmetic Series – The sum of the terms that form an arithmetic sequence.

FORMULAS:

Sn = n/2[2t1 + (n-1)d]

Sn  = the sum of the terms in the arithmetic sequence.

OR

Sn = n/2(t1 + tn)

Examples:

1. Read page 25 Example 1:
   1. Determine the total number of flashes for the male firefly in 42 min.
2. The sum of the first two terms of an arithmetic series is 19 and the sum of the first four terms is 50. What are the first six terms of the series and the sum to 20 terms?

**1.3 – Geometric Sequence**

Geometric Sequence – a sequence of numbers where you multiply by a common ratio to ge the next number in the sequence.

Common Ratio – 2nd number divided by 1st number

How do we solve: n3 = 8 OR 5x = 125

FORMULA:

tn = t1rn-1

tn = the general term or nth term

t1 = first term

r = common ratio

n = number of terms

Examples:

1. Suppose there were three bacteria originally present in a sample. Determine the general term that relates the number of bacteria to the doubling period of the bacteria. State the values of t1 and r in the geometric sequence formed.
2. Suppose the smallest reduction a photocopier could make is 60% of the original. What is the shortest possible length after 8 reductions of a photograph that is originally 42 cm long?
3. In a geometric sequence, the second term is 28 and the fifth term is 1792. Determine the values of t1 and r and list the first three terms of the sequence.
4. In 1990 the population of Canada was approximately 26.6 million. The population projection for 2025 is approximately 38.4 million. If this projection were based on a geometric sequence, what would be the annual growth rate? Given that this is a geometric sequence what assumptions would you have to make?

**1.4 – Geometric Series**

Geometric Series – The sum of the terms of a geometric sequence

FORMULA:

Sn =

Examples:

1. Determine the sum of the first 8 terms of the following geometric sequences:
   1. 5+15+45+…
   2. t1 = 64, r = ¼
   3. 1/64 + 1/16 + ¼ + … +1024
   4. -2+4-8+…--8192

**1.5 – Infinite Geometric Series**

Infinite Geometric Series – Finding the sum of a geometric series that goes to infinite.

Convergent series – a series with an infinite number of terms, in the sum approaches a specific value. 1 + ½ + ¼ + 1/8+…

Divergent series – A series with an infinite number of terms that does not approach a specific number. 2+4+8+16+32+…

FORMULA:

The common ratio must be between -1 and 1.

Examples:

1. Determine where each infinite geometric series is convergent or divergent. Calculate the sum, if it exists:
   1. 1 + 1/5 + 1/25 + …
   2. 4+8+16+…
2. You can express 0.584584584… as an infinite geometric series, determine the sum of the series.