**6.1 – Rational Expressions**

Rational Expression – a fraction that includes variables. Has a polynomial in its numerator and denominator.

Non-permissible Value – your denominator can not be 0. Your non permissible value is a value that makes your denominator = 0. Get this number by setting all values in your denominator = 0.

Examples:

1. Determine the non-permissible value(s) for each rational expression:
	1. $\frac{4a}{3bc}$ b. $\frac{x-1}{\left(x+2\right)(x-3)}$ c. $\frac{2y^{2}}{y^{2}-4}$

Simplifying Rational expressions:

1. Factor the numerator and denominator, if possible
2. Cross out all factors that are the same, reduce numbers.
3. State the NPV

Examples:

1. Simplify each rational expression. What is the non-permissible value(s)?
	1. $\frac{2y^{2}+ y-10}{y^{2}+ 3y-10}$ b. $\frac{6-2m}{m^{2}- 9}$
2. Use the rational expression $\frac{16m^{2}- 9y^{2}}{8x-6y}$ to help answer the following:
	1. What is the non-permissible value for y if x = 3?
	2. Evaluate the expression for x = 1.5 and y = 2.8.
	3. Give a reason why it may be beneficial to simplify a rational expression.

**6.1 Assignment: Page 317 #3 ace, 4 acef, 6, 8,11,13**

6.2 – Multiplying and Dividing Rational Expressions

Multiplying Rational Expressions:

* + 1. Factor all polynomials, if possible.
		2. State the NPV of ALL denominators
		3. Cross off any factors that are the same (any from the top with any from the bottom)

Dividing:

1. Flip the rational expression after the division sign, change to multiply
2. Follow multiplying rules
3. NPV find before you flip and after you flip

Examples:

1. Express each product in simplest form. What are the non-permissible values?
	1. $\frac{d}{2πr}x\frac{2πrh}{d-2}$
	2. $\frac{y^{2}- 9}{r^{3}- r} x \frac{r^{2}- r}{y+3}$
	3. $\frac{a^{2}- a-12}{a^{2}- 9}$ x $\frac{a^{2}- 4a+3}{a^{2}-4a}$
2. Simplify. What are the non-permissible values?
	1. $\frac{c^{2}- 6c-7}{c^{2}- 49}÷\frac{c^{2}+ 8c+7}{c^{2}+ 7c}$
	2. $\frac{3x+12}{3x^{2}- 5x-12}÷\frac{12}{3x+4}x\frac{2x-6}{x+4}$

6.3- Adding and subtracting Rational Expressions

Adding/Subtracting:

1. Factor all of the denominators
2. Find the LCD by attached all of the factors from #1 (remember what you do to the bottom you must do to the top)
3. + or – the numerators
4. Find the NPV
5. Simplify, if possible

Examples:

1. Determine each sum or difference. Express each answer in simplest form. Identify all non-permissible values.
	1. $\frac{m}{n}- \frac{m+1}{n}$
	2. $\frac{10m-1}{4m-3}- \frac{8-2m}{4m-3}$
	3. $\frac{2x^{2}- x}{\left(x-3\right)(x+1)}+ \frac{3-6x}{\left(x-3\right)(x+1)}- \frac{8}{\left(x-3\right)(x+1)}$
	4. $\frac{4}{p^{2}- 1}+ \frac{3}{p+1}$
	5. $\frac{x-1}{x^{2}+ x-6}- \frac{x-2}{x^{2}+ 4x+3}$
	6. $\frac{2- \frac{4}{y}}{y- \frac{4}{y}}$

EXTRA EXAMPLES:

***6.4 – Rational Equations***

These have = signs.

1. Factor all denominators
2. X each term by the LCM (This is the same as LCD)
3. Solve for x.

Examples:

1. Solve. What are the non-permissible values?
	1. $\frac{9}{y-3}- \frac{4}{y-6}= \frac{18}{y^{2}- 9y+18}$
	2. $\frac{3x}{x+2}- \frac{5}{x-3}= \frac{-25}{x^{2}- x-6}$
2. Stella takes 4 h to paint a room. It takes Jose 3 h to paint the same area. How long will the paint job take if they work together?
3. A train has a scheduled run of 160 km between two cities in Saskatchewan. If the average speed is decreased by 16km/h, the run will take 1/2h longer. What is the average speed of the train?

Remember: D = RT Or T = $\frac{D}{R}$

|  |  |  |
| --- | --- | --- |
| ***Distance*** | ***Rate*** | ***Time***  |
|   |   |   |
|   |   |   |

**7.4 Reciprocal Functions (Day 1)**

**Reciprocal:** (A number) x (its reciprocal) = 1. For any non-zero number “”, the reciprocal of  is **Reciprocal Function**: For any function , its reciprocal is, provided that 

Example: Sketch the graphs of  and  on the same set of axes.
  

|  |  |
| --- | --- |
| ***x*** | ***y*** |
| -10 |  |
| -5 |  |
| -2 |  |
| -1 |  |
| -1/2 |  |
| -1/5 |  |
| -1/10 |  |
| 0 |  |
| 1/10 |  |
| 1/5 |  |
| ½ |  |
| 1 |  |
| 2 |  |
| 5 |  |
| 10 |  |

|  |  |
| --- | --- |
| ***x*** | ***y*** |
| -10 |  |
| -5 |  |
| -2 |  |
| -1 |  |
| -1/2 |  |
| -1/5 |  |
| -1/10 |  |
| 0 |  |
| 1/10 |  |
| 1/5 |  |
| ½ |  |
| 1 |  |
| 2 |  |
| 5 |  |
| 10 |  |

|  |  |  |
| --- | --- | --- |
| **Characteristic** |  |  |
| **Domain** |  |  |
| **Range** |  |  |
| **End Behaviour** | As *x* becomes a very “large negative value”, *y* becomes aAs *x* becomes a very “large positive value”, *y* becomes a | As *x* becomes a very “large negative value”, *y* becomes aAs *x* becomes a very “large positive value”, *y* becomes a |
| **Behaviour at x = 0**  |  |  |
| **Invariant Points of and**  |  |

The function  is a rational function that has 2 distinct branches located on either side of the vertical asymptote (non-permissible value(s) of x)

**Asymptote** – a line that a curve approaches.

**Vertical Asymptotes** – occurs at the non-permissible values of the domain( x –value) of the rational function.

**Horizontal Asymptotes** – occurs when y= 0 because 
Given , then the reciprocal is . To graph the reciprocal:

1. Find the vertical asymptote (the non-permissible values of the reciprocal function). It
will occur at the x-intercept ( zero) of the original function.

2. Find the invariant points. Set *y =* ±1 and solve for *x*.

3. Find the intercepts.

**Example 1**

**Graph y = 2x + 5 and its reciprocal**
a) Write the reciprocal function:

b) Determine the equation of the vertical asymptote of the reciprocal function:

e) Sketch the graph of the original and reciprocal function

c) Determine the Invariant points at y = 1 and y = -1

d) Find the x and y intercept of the reciprocal function

**Example 2**

**Graph  and its reciprocal.**

a) Find the zeros (x-intercept) of the f(x).( original function)

**b)** Determine its reciprocal function .

**c)** Determine the equation of the vertical asymptote of the reciprocal function. How are the zeros ( x-intercept) of the original function related to the non-permissible values?

d) Sketch the graph of the reciprocal function

Assignment: Page 403 #1ab, 2ab, 3ab, 5ab, 7abc